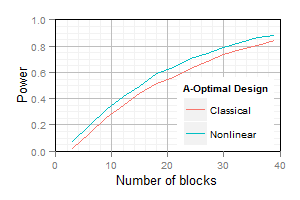
# 2A. ABSTRACT OF RESEARCH PROPOSAL

The foundations of modern experimental design were laid in 1918 with the introduction of the ANOVA models1. They are conditional on *randomization*, for external validity, *replication*, to estimate error variances, and *blocking*, to control *efficiency* (i.e. parameter estimators’ variances). Blocking’s effectiveness is optimal in a *complete* block arrangement where each *treatment* (i.e. experimental condition) occurs equally frequently in each block2. However, naturally occurring groupings of homogeneous experimental units often form *incomplete* blocks3 (IBs) at the cost of loss of efficiency. The challenge is to find an *optimal* arrangement of treatments across blocks, i.e. having maximum efficiency among a competing set of designs of the same size.

Classical solutions to finding optimal designs are premised on the assumption of unit-treatment *additivity*, a relic of ANOVA models, enabling *global* searches across the entire competing design-set. Optimality criteria yielding designs having all treatment means estimated with equal precision are typically used, resulting in the ‘holy grail’ class of *balanced* incomplete block (BIB) designs (i.e. all treatments equally replicated and pairs of treatments concurring equally often among blocks)5. *But is this notion of balance meaningful when additivity is violated*, e.g. when the data-generating distribution (d.g.d) is *non-linear*?

Our preliminary work suggests that the classical notion of balance does not carry over to the non-linear setting. E.g., consider an experiment with treatments *A*, *B* and *C* arranged in three blocks each with two units, where the d.g.d. is (). Classical theory holds that the BIB design is globally *A*-optimal4 (i.e. minimum average variance). However, an exhaustive search of all competing designs, accounting for the magnitudes of the treatment means , and , yields the *locally* *A*-optimal design in which the treatments’ replications are inversely related to the relative magnitudes of their means. A power analysis of 10,000 simulated experiments (**Figure**) of locally *A*-optimal designs accounting for the additivity breakdown, for 3 to 39 blocks, shows these designs always outperform the globally *A*-optimal designs constructed from replicate sets of . Furthermore, the *A*-optimal non-linear design achieves 80% power with 20% less experimental effort than the classical *A*-optimal design. This small example clearly demonstrates that *the theory of* *optimal designs in the non-linear setting must be built on post-modern foundations, without assuming additivity, to enable the development of the range of classes of designs that exist in the classical setting*.

Given that in 1972 generalized linear models5 availed us of a unified theory to accommodate non-linear d.g.d.’s, it is surprising that work in this area only began in 20066. This has to date focused exclusively on response surface models7–9 where the values of the treatment factor levels (on ) are themselves integral to the optimality criteria. In contrast, we consider block designs in which the values of the factor levels play *no* role in design optimality because they are fixed at the outset.

*Our primary goal is* to lay the theoretical foundations that will enable the efficient construction and analysis of designs for single- and multi-factor experiments for non-linear d.g.d is. More specifically, we will:

1. Explore the performances and relative merits of different *objective functions* based on *A*-, *D*- (i.e. generalised variance) and *E*-optimality (i.e. minimax of individual contrasts) criteria10,
2. Explore the properties of the optimal designs generated by these criteria to identify families of designs for different blocking structures (e.g. complete, incomplete and row-column designs) to:
   * Develop a general theory for their construction or, when analytical optimisation of our objective functions is not possible,
   * Develop computer algorithms using *stochastic optimisation methods*11-14.
3. Explore how the methods we develop may lead to a unified framework for generating optimal block designs for the exponential family of d.g.d.’s.

Our team has published in top-tiered methodological and application journals15-20[Chris’ stuff to be added] on the optimal design of single- and multi-factor experiments spanning both the additivity and non-linear paradigms. Our extensive experience in actually designing experiments across a broad range of disciplines provides us with the essential insights needed to identify the opportunities and challenges present when implementing designed experiments in practice.

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